Week 1: Implications

Some of you were (understandably) confused by the truth table for the implication sign:

P	$\implies$	Q
T	**T**	T
T	**F**	F
F	**T**	T
F	**T**	F

When P is true, it is fairly clear what to do, as the intended meaning is that Q should be true as a result. So when P is true, Q must be true as well, so we return T for the implication sign when P and Q are both true, and F when P is true and Q is false. The situation when P is false is less clear, but hopefully is well illustrated by the following example, which I showed to a couple of you during the tutorial. Hopefully everybody can agree that the statement:

$$(x > 2) \implies (x^2 > 4)$$

is true. So whatever the values of x, when we apply the above truth table, we should always see T as the result. If for example we set $x = 3$, then we find that $x > 2$ is true, as is $x^2 > 4$, so we are in the first row of the truth table, and indeed the result of the implication is T. If however we set $x = -3$, then we find that $x > 2$ is false, but $x^2 > 4$ is still true. This is why we define the implication sign to be T in the third row of the table. Finally, if we set $x = 0$, then we find that $x > 2$ is false, and $x^2 > 4$ is false as well, which explains why we must have T in the final row of the truth table.

Note that it is impossible to pick an x so that we end up in the second row, i.e. an x such that $x > 2$ but $x^2 \leq 4$.

One simple trick to help you understand statements involving implication signs is to change how you read them; try resisting the temptation to read $A \implies B$ as "A implies B", and instead read it as "if A is true then B is true", or "if A then B" for short.